Universidad
Rey Juan Carlos

DEGREE IN AEROSPACE ENGINEERING IN AIR NAVIGATION

Collection of exams with solutions
Course on Aerospace Engineering

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Contents

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Partial exam

Partial Exam, 27th March, 2012

1.1 Theory

Theory: 1 hour and 30 minutes; 6 points out of 10.

Exposition questions: **Expose and develop the posed questions. Use additional paper to assess the questions. In order to have an quantitative measured, the estimated times and spaces are given.**

1. International Standard Atmosphere:

   (a) Deduce the equations of the International Standard Atmosphere (ISA) in the troposphere $(0 \leq h \leq 11000\ [m])$: [**0.75 pt**]

   - Expose the considered hypotheses.
   - Deduce the equations that relate pressure and density with altitude.

   (b) Calculate pressure and density at an altitude of $h = 10000$ m considering ISA+10. [**0.25 pt**]

   [**Estimated time: 20 min; Estimated space: 1 paper side**]

2. How lift is produced?

   (a) Continuity equation: [**0.2 pt**]

   - Expose the considered hypotheses.
   - State the continuity equation.

   (b) Bernoulli equation: [**0.4 pt**]

   - Expose the considered hypotheses.
   - Deduce Bernoulli equation considering that gravity effects do not have influence.

   (c) Based on both continuity and Bernoulli equations, explain how lift is produced. Draw an airfoil sketch with a typical distribution of the coefficient of pressures over the airfoil. [**0.4 pt**]

   [**Estimated time: 20 min; Estimated space: 1 paper side**]

3. Define the different weights of an aircraft. Define also the limitations on the weights of an aircraft. Draw a sketch of a typical Payload-Diagram, pointing out the different weights and limitations. Analyze how the range of an aircraft evolves depending on payload and fuel weight. [**1 pt**]

   [**Estimated time: 15 min; Estimated space: 1 paper side**]

Short questions: **Brief statement of the posed questions. It is compulsory to use the space given bellow the statements to assess the questions.**

4. State the main challenges aviation is facing: [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

5. State the structural elements of a typical wing and comment briefly their main structural functions: [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

6. Explain briefly what the flight-by-wire system is and what are its main advantages: [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

Test: The test contains 15 questions, each with 3 options. Mark the options you think are correct. There might be non, one, two or three correct options. If you have marked all (in case there is at least one) the correct option/s, you get 0.1 points. If you do not mark all the correct option/s (in case there is at least one), you get 0 points. There might be options in which none of the options is correct. In such a case you shouldn't mark any to get 0.1 points.

Marked on blue are the correct answers:

1. Which of the following are fundamental characteristics of the aerospace industry: [**0.1 pt**]
   (a) Specific technologies in the vanguard which spin-out to other sectors.
   (b) Limited series (non mass production) and difficult automation of manufacturing processes.
   (c) Long term development of new projects.
2. Which of the following are main control surfaces of the aircraft: [**0.1 pt**]
   (a) Flaps.
   (b) Spoilers.
   (c) Ailerons.
3. Which of the following are inertial or quasi-inertial reference frames: [**0.1 pt**]
   (a) Earth reference frame.
   (b) Local Horizon frame.
   (c) Wind axis frame.
4. Regarding the boundary layer, mark those sentences that are true among the following: [**0.1 pt**]
   (a) It is a thin layer, in which the velocity perpendicular to the airfoil varies dramatically.
   (b) Inside the boundary layer, viscosity can be neglected without significant effects on the solution of Navier-Stokes equation.
   (c) In high-performance designs, such as commercial transport aircraft, much attention is paid to controlling the behavior of the boundary layer to minimize drag.

5. Regarding the Reynolds number, mark those sentences that are true among the following: [**0.1 pt**]

   (a) The Reynolds number Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for a given flow conditions.

   (b) $Re = \frac{\mu V D}{\rho}$, where Re is the Reynolds number, μ is the dynamic viscosity of the fluid, V is the mean velocity of the fluid, D is the characteristic length in the study, and ρ is the density of the fluid.

   (c) In flight, the Reynolds number is high, of an order of millions, and the flow is typically laminar.

6. Which of the following are considered characteristic curves of an airfoil: [**0.1 pt**]

   (a) $c_l(\alpha)$.

   (b) $c_l(c_d)$.

   (c) $c_m(\alpha)$.

7. Which of the following are considered good strategies to retard the appearance of divergency: [**0.1 pt**]

   (a) Increase the relative thickness.

   (b) Design supercritical airfoils.

   (c) Introduce wing swept.

8. Regarding the materials used in aircraft, mark those sentences that are true among the following: [**0.1 pt**]

   (a) The pure aluminum has a relatively low strength, it is a extremely flexible metal with virtually no structural applications. Alloyed with different metals improves its properties, and thus aluminum alloys have been extensively used in airframes and skins.

   (b) Steels are typically used in landing gear pivot brackets and wing tips.

   (c) Composite materials consist of strong fibres such as glass or carbon set in a matrix of plastic or epoxy resin.

9. In a typical T-arrangement layout for the instruments: [**0.1 pt**]
   (a) The airspeed indicator is located on top-right.
   (b) The altimeter is located on top-left.
   (c) The heading indicator is located on top-center.
10. A typical jet configuration is as follows: [**0.1 pt**]
   (a) Intake-Compressor-Burner-Turbine-After burner-Nozzle.
   (b) Intake-Compressor-Burner-Turbine-Nozzle-After burner.
   (c) Intake-Compressor-Turbine-Burner-After burner-Nozzle.
11. The compressor pressure ratio (CPR) can be defined as: [**0.1 pt**]
   (a) $CPR = \frac{p_{3t}}{p_{2t}}$.
   (b) $CPR = (\frac{T_{3t}}{T_{2t}})^{\frac{\gamma}{\gamma-1}}$.
   (c) $CPR = \frac{p_{2t}}{p_{3t}}$.
12. Regarding turbofans, mark those sentences that are true among the following: [**0.1 pt**]
   (a) A turbofan engine is a variation of the basic gas turbine engine, where the core engine is surrounded by a fan in the front and an additional fan turbine at the rear.
   (b) A turbofan operates more efficiently than a propeller at low speeds.
   (c) The ration between the air mass that flows around the engine and the air mass that goes through the core is called the bypass ratio.
13. The maximum aerodynamic efficiency can be defined as: [**0.1 pt**]
   (a) $E_{max} = \sqrt{\frac{C_{D_0}}{C_{D_i}}}$.
   (b) $E_{max} = \frac{1}{2\sqrt{C_{D_0} C_{D_i}}}$.
   (c) $E_{max} = \sqrt{\frac{C_{D_i}}{C_{D_0}}}$.

14. Regarding control theory, mark those sentences that are true among the following: [**0.1 pt**]

    (a) The usual objective of control theory is to calculate solutions for the proper corrective action from the controller that result in system stability, that is, the system will hold the set point and not oscillate around it.

    (b) The input and output of the system are related to each other by what is known as a transfer function.

    (c) The system is defined by a system of differential equations.

15. An aircraft is said to be statically stable if it is fulfilled that: [**0.1 pt**]

    (a) $C_{M,cg} > 0$.

    (b) $C_{M,\alpha} < 0$.

    (c) $C_{M,\delta_e} < 0$.

1.2 Problems

Problems: 1 hour; 4 points out of 10.

Problem 1 [2 pt] [Estimated time: 30 min]

Consider an Airbus A-320 with the following characteristics:

- $m = 64$ tonnes
- $S_w = 122.6\ m^2$
- $C_D = 0.024 + 0.0375 C_L^2$

1. The aircraft starts an ascent maneuver with uniform velocity at 10.000 feet of altitude (3048 meters). At that flight level, the typical performances of the aircraft indicate a velocity with respect to air of 289 knots (148.67 m/s) and a rate of climb (vertical velocity) of 2760 feet/min (14 m/s). Assuming that $\gamma << 1$, calculate:
   (a) The angle of ascent, γ. [**0.25 pt**]
   (b) Required thrust at those conditions. [**0.25 pt**]
2. The aircraft reaches an altitude of 11000 m, and performs a horizontal, steady, straight flight. Determine:
   (a) The velocity corresponding to the maximum aerodynamic efficiency. [**0.75 pt**]
3. The pilot switches off the engines and starts gliding at an altitude of 11000 m. Calculate:
   (a) The minimum descent velocity (vertical velocity), and the corresponding angle of descent, γ_d. [**0.75 pt**]

Solution to Problem 1

Besides the data given in the statement, the following data have been used:

- $g = 9.81\ m/s^2$
- R= 287 J/(kgK)
- $\alpha_T = 6.5 \cdot 10^{-3}$ K/m
- $\rho_0 = 1.225\ kg/m^3$
- $T_0 = 288.15$ K
- ISA: $\rho = \rho_0 (1 - \frac{\alpha_T h}{T_0})^{\frac{gR}{\alpha_T} - 1}$

1. Unifrom-ascent under the following flight conditions:

- h=2048 m. Using ISA $\rightarrow \rho = 0.904\ kg/m^3$
- V= 148.67 m/s.
- $\dot{h}_e = 14$ m/s.

The system that governs the motion of the aircraft is:

$$T = D + mg \sin \gamma, \tag{1.1a}$$
$$L = mg \cos \gamma, \tag{1.1b}$$
$$\dot{x}_e = V \cos \gamma, \tag{1.1c}$$
$$\dot{h}_e = V \sin \gamma, \tag{1.1d}$$

Assuming that $\gamma << 1$, and thus that $\cos \gamma \sim 1$ and $\sin \gamma \sim \gamma$, System (1.1) becomes:

$$T = D + mg\gamma, \tag{1.2a}$$
$$L = mg, \tag{1.2b}$$
$$\dot{x}_e = V, \tag{1.2c}$$
$$\dot{h}_e = V\gamma, \tag{1.2d}$$

a) From Equation (1.2d), $\gamma = \frac{\dot{h}_e}{V} = 0.094\ rad$ (5.39°).

b) From Equation (1.2a), $T = D + mg\gamma$.

$$D = C_D \frac{1}{2} \rho S_w V^2, \tag{1.3}$$

where $C_D = 0.024 + 0.0375C_L^2$, and ρ, S_w, V^2 are known.

$$C_L = \frac{L}{\frac{1}{2}\rho S_w V^2} = 0.512, \quad (1.4)$$

where, according to Equation (1.2b), $L = mg$. With Equation (1.4) in Equation (1.3), $D = 41398$ N.

Finally:

$$T = D + mg\gamma = 100\ kN$$

2. Horizontal, steady, straight flight under the following flight conditions:

- h=11000 m. Using ISA $\rightarrow \rho = 0.3636\ kg/m^3$
- The aerodynamic efficiency is maximum.

The system that governs the motion of the aircraft is:

$$T = D, \quad (1.5a)$$
$$L = mg\cos\gamma, \quad (1.5b)$$

The maximum Efficiency is $E_{max} = \frac{1}{2\sqrt{C_{D_0}C_{D_i}}} = 16.66$.

The optimal coefficient of lift is $C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{C_{D_i}}} = 0.8$.

$$C_L = \frac{L}{\frac{1}{2}\rho S_w V^2} \rightarrow V = \sqrt{\frac{L}{\frac{1}{2}\rho S_w C_L}} = 187\ m/s,$$

where, according to Equation (3.12b), $L = mg$, and in order to fly with maximum efficiency: $C_L = C_{L_{opt}}$.

3. Gliding under the following flight conditions:

- h=11000 m. Using ISA $\rightarrow \rho = 0.3636\ kg/m^3$
- At the minimum descent velocity.

The system that governs the motion of the aircraft is:

$$D = mg\sin\gamma_d, \quad (1.6a)$$
$$L = mg\cos\gamma_d, \quad (1.6b)$$
$$\dot{x}_e = V\cos\gamma_d, \quad (1.6c)$$
$$\dot{h}_{e_{des}} = V\sin\gamma_d, \quad (1.6d)$$

Notice that $\gamma_d = -\gamma$.

Assuming that $\gamma_d << 1$, and thus that $\cos\gamma_d \sim 1$ and $\sin\gamma_d \sim \gamma_d$, System (1.6) becomes:

$$D = mg\gamma_d, \tag{1.7a}$$
$$L = mg, \tag{1.7b}$$
$$\dot{x}_e = V, \tag{1.7c}$$
$$\dot{h}_{e_{des}} = V\gamma_d, \tag{1.7d}$$

In order to fly with maximum descent velocity $\dot{h}_{e_{des}}$ must be maximum. Operating with Equation (1.7a) and Equation (1.7b), $\gamma_d = \frac{D}{L}$.

$$\dot{h}_{e_{des}} = V\gamma = V\frac{D}{L} = V\frac{(0.024 + 0.0375C_L^2)\frac{1}{2}\rho S_w V^2}{C_L\frac{1}{2}\rho S_w V^2}. \tag{1.8}$$

Knowing that $C_L = \frac{L}{\frac{1}{2}\rho S_w V^2}$, where, according to Equation (1.7b), $L = mg$, Equation (1.8) becomes:

$$\dot{h}_{e_{des}} = \frac{V}{mg}\left(0.024\frac{1}{2}\rho S_w V^2 + \frac{0.0375(mg)^2}{\frac{1}{2}\rho S_w V^2}\right). \tag{1.9}$$

Make $\frac{\partial \dot{h}_{e_{des}}}{\partial V} = 0$.

The velocity with respect to air so that the vertical velocity is minimum is:

$$V = \sqrt[4]{\frac{4}{3}\frac{C_{D_i}}{C_{D_0}}}\sqrt{\frac{mg}{\rho S_w}} = 142.57\ m/s \tag{1.10}$$

Substituting V=142.57 m/s in Equation (1.9), $\dot{h}_{e_{des}} = 9.87$ m/s.

Problem 2 [2 pt] [Estimated time: 30 min]

1. In a wind tunnel experiment it has been measured the distribution of pressures over a symmetric airfoil for an angle of attack of 14°. The distribution of coefficient of pressures at the intrados, C_{pI}, and extrados, C_{pE}, of the airfoil can be respectively approximated by the following functions:

$$C_{pI}(x) = \begin{cases} 1 - 2\frac{x}{c}, & 0 \leq x \leq \frac{c}{4}, \\ \frac{2}{3}(1 - \frac{x}{c}) & \frac{c}{4} \leq x \leq c; \end{cases}$$

$$C_{pE}(x) = \begin{cases} -12\frac{x}{c}, & 0 \leq x \leq \frac{c}{4}, \\ 4(-1 + \frac{x}{c}) & \frac{c}{4} \leq x \leq c. \end{cases}$$

   (a) Draw the curve that represents the distribution of pressures. [**0.25 pt**]

   (b) Considering a chord $c = 1$ m, obtain the coefficient of lift of the airfoil. [**0.75 pt**]

   (c) Calculate the slope of the characteristic curve $c_l(\alpha)$. [**0.25 pt**]

2. Based on such airfoil as cross section, we build a rectangular wing with a wingspan of $b = 20$ m and constant chord $c = 1$ m. The distribution of the coefficient of lift along the wingspan of the wing (y axis) for an angle of attack $\alpha = 14°$ is approximated by the following parabolic function:

$$c_l(y) = 1.25 - 5(\frac{y}{b})^2, \quad -\frac{b}{2} \leq y \leq \frac{b}{2}.$$

   (a) Draw the curve $c_l(y)$. [**0.25 pt**]

   (b) Calculate the coefficient of lift of the wing. [**0.5 pt**]

Solution to Problem 2

1. Airfoil:

a) The curve is as follows:

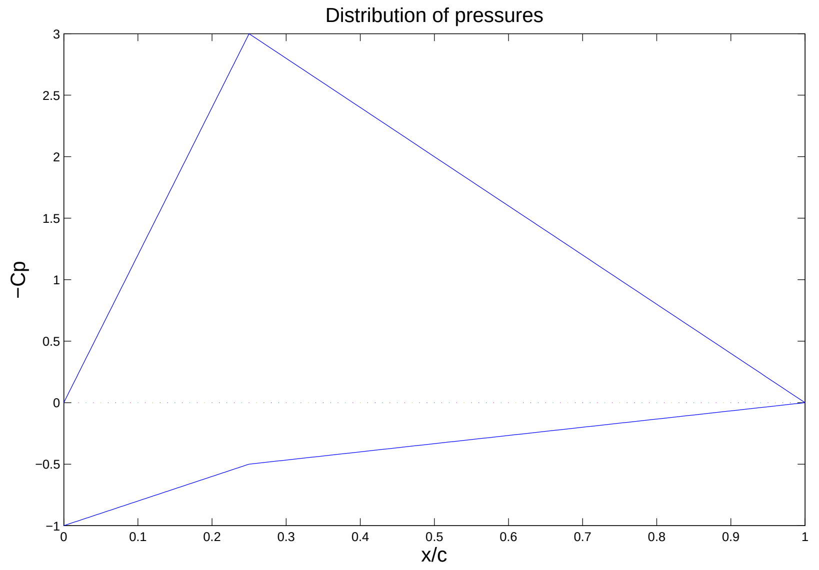


Figure 1.1: Distribution of pressures

b) The coefficient of lift of the airfoil for $\alpha = 14°$can be calculated as follows:

$$c_l = \frac{1}{c}\int_{x_{le}}^{x_{te}} (c_{pI}(x) - c_{pE}(x))dx, \tag{1.11}$$

In this case, with $c = 1$ and the given distributions of pressures of Intrados and extrados, Equation (1.11) becomes:

$$c_l = \frac{1}{c}\left[\int_0^{1/4} \big((1-2x)-(-12x)\big)dx + \int_{1/4}^{1} \big(2/3(1-x)-4(-1+x)\big)dx\right] = 1.875.$$

c) The characteristic curve is given by:

$$c_l = c_{l_0} + c_{l_\alpha}\alpha.$$

Since the airfoil is symmetric: $c_{l_0} = 0$. Therefore $c_{l_\alpha} = \frac{c_l}{\alpha} = \frac{1.875 \cdot 360}{14 \cdot 2\pi} = 7.16$ 1/rad.

1. Wing:

a) The curve is as in Figure 3.3:

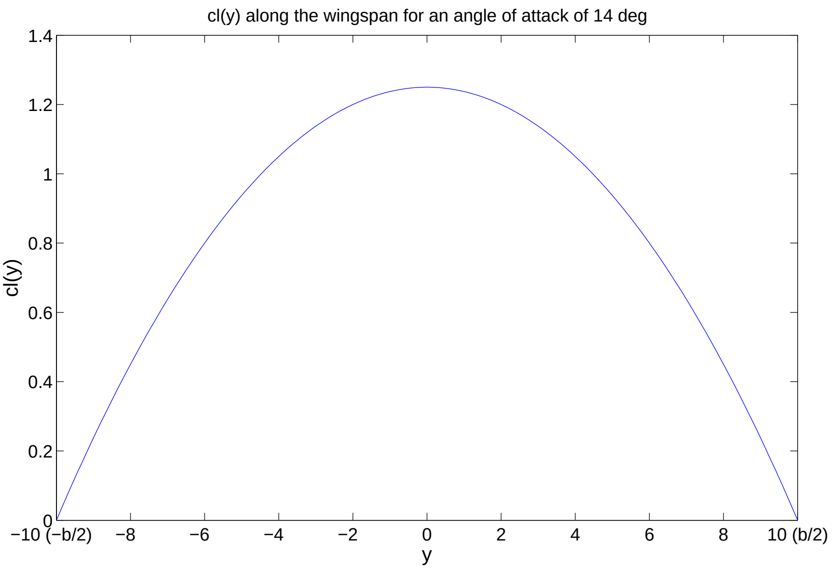


Figure 1.2: Curve $c_l(y)$

b) The coefficient of lift for the wing for $\alpha = 14°$can be calculated as follows:

$$C_L = \frac{1}{S_w}\int_{-b/2}^{b/2} c(y)c_l(y)dy. \tag{1.12}$$

Substituting in Equation (1.12) considering c(y)=1 and b=20:

$$C_L = \frac{1}{20}\int_{-10}^{10}\left(1.25 - 5(\frac{y}{20})^2\right)dy = 0.83.$$

2

Final exam

Final Exam, 18th May, 2012

2.1 Theory

Theory: 1 hour and 40 minutes; 6 points out of 10.

Exposition questions: **Expose and develop the posed questions. Use additional paper to assess the questions. In order to have an quantitative measure, the estimated times and spaces are given.**

1. The air navigation system:

   (a) Define what the air navigation system is, state what is its main goal. [**0.2 pt**]

   (b) According to Figure 2.1, state the main phases of a flight. [**0.1 pt**]

   (c) Define what the Air Traffic Management System is. [**0.1 pt**]

   (d) How the different levels of Air Traffic Management affect a typical flight such the one in Figure 2.1? [**0.2 pt**]

   (e) Throughout which volumes of responsibility does an aircraft overflies during a flight such the one in Figure 2.1? [**0.2 pt**]

   (f) What kind of Air Transit Services must be provided according to the different phases of a typical flight such the one in Figure 2.1? [**0.2 pt**]

   [**Estimated time: 30 min; Estimated space: 1-2 paper sides**]

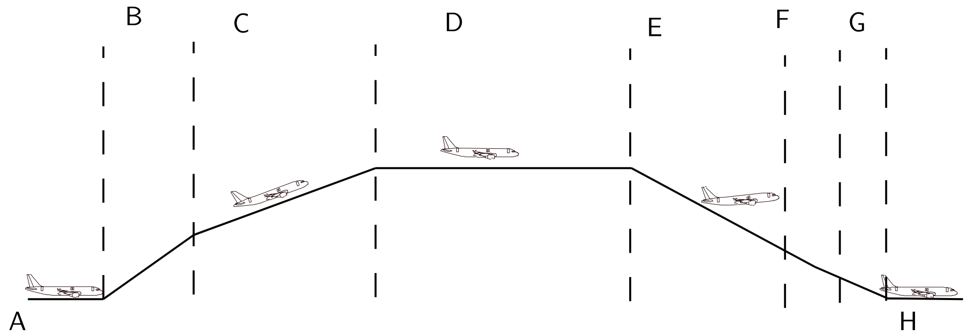


Figure 2.1: Phases in a flight.

2. SESAR:

   (a) Expose the fundamental challenges the current ATM system is facing. [**0.2 pt**]

   (b) State the main goals of SESAR. [**0.2 pt**]

   (c) Expose the key features of the 2020 ATM Target Concept. [**0.6 pt**]

   [**Estimated time: 15 min; Estimated space: 1 paper side**]

3. Regarding the range of an aircraft,

   (a) Deduce the **Breguet Equation**, pointing out the considered hypotheses. In particular, consider two cases:

      i. Constant velocity and constant aerodynamic efficiency[1]. [**0.3 pt**]

      ii. Varying velocity (Velocity as a function of mass) and constant aerodynamic efficiency. [**0.3 pt**]

   (b) Draw a sketch of a typical Payload-Diagram. [**0.2 pt**]

   (c) Making use of the deduced Breguet equation and the Payload diagram, discuss how weight affects range in a typical commercial aircraft. [**0.2 pt**]

   [**Estimated time: 20 min; Estimated space: 1 paper side**]

[1] Indeed, properly speaking, the Breguet Equation is that resulting of this case.

Solution to 3.b.ii) [0.3 pt].

Considering that the aircraft performs a linear,horizontal, steady flight, we have that:

$$L = mg \tag{2.1}$$
$$T = D \tag{2.2}$$
$$\dot{x} = V \tag{2.3}$$
$$\dot{m} = -\eta T \tag{2.4}$$

Since $\dot{x} = \frac{dx}{dt}$, it is clear that the Range, R, looking at Equation (2.3), can be expressed as:

$$R = \int_{t_i}^{t_f} V dt. \tag{2.5}$$

Now, since $\dot{m} = \frac{dm}{dt} = -\eta T$, Equation (2.5) yields:

$$R = \int_{m_i}^{m_f} -\frac{V}{\eta T} dm, \tag{2.6}$$

where m_i is the initial mass and m_f is the final mass.

Using Equation (2.23) and Equation (2.24), Equation (2.6) yields:

$$R = \int_{m_i}^{m_f} -\frac{VE}{\eta g}\frac{dm}{m}. \tag{2.7}$$

with $E = \frac{L}{D}$.

Assuming that V and E (also η and g) are constant, we reach to the well-known Breguet equation (point i)). However, If we assume that the speed depends on the mass of the aircraft, we have that:

$$V = \sqrt{\frac{2L}{\rho S c_l}} = \sqrt{\frac{2mg}{\rho S c_l}}, \tag{2.8}$$

and

$$R = \int_{m_i}^{m_f} -\sqrt{\frac{2g}{\rho S c_l}}\frac{E}{\eta g}\frac{dm}{\sqrt{m}}. \tag{2.9}$$

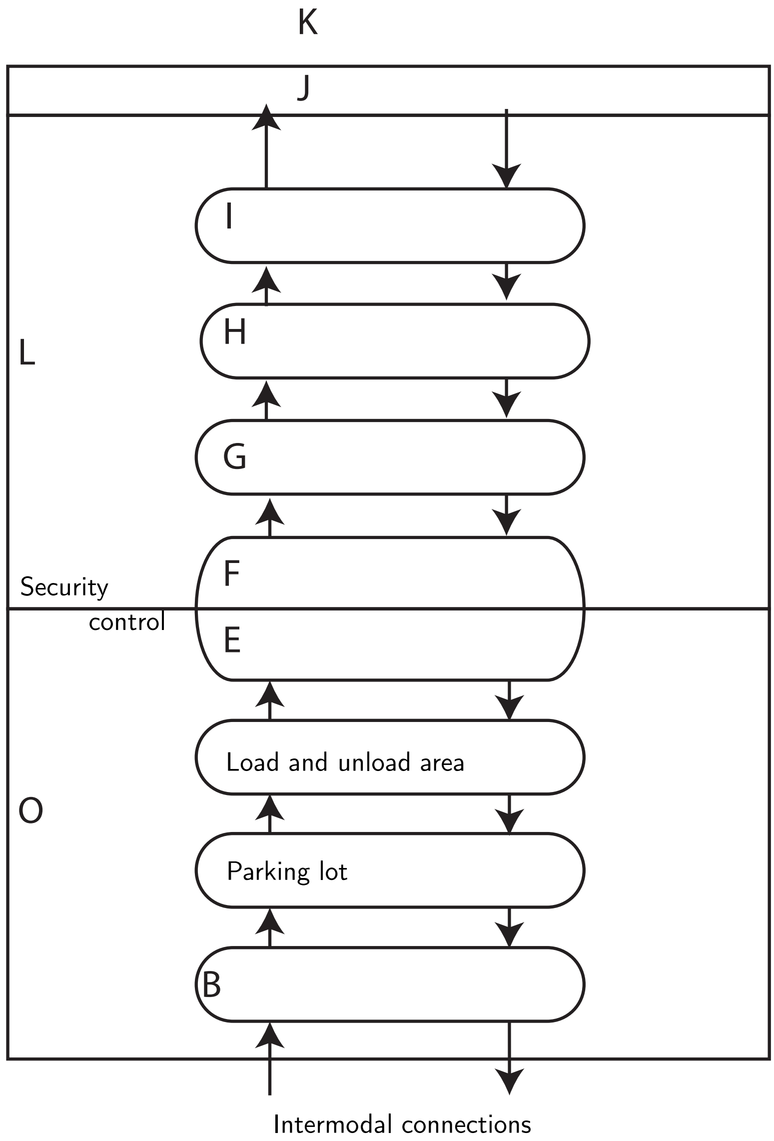
Assuming now that c_l, c_d (therefore E), g, ρ and η, and integrating Equation 2.9:

$$R = \sqrt{\frac{2}{\rho S c_l g}}\frac{E}{\eta}(\sqrt{m_i} - \sqrt{m_f}). \tag{2.10}$$

Short questions: **Brief statement of the posed questions. It is compulsory to use the space given bellow the statements to assess the questions.**

4. Regarding the schematic configuration of a modern airport, fill in the gaps [letter B, E, F, etc.] in the figure given bellow: [**0.5 pt**]

   [**Estimated time: 2 min; Complete it in the figure given below**]



5. According to the classical mechanics, to orientate without loss of generality a reference frame system F_I with respect to another F_F: if both have common origin, it is necessary to perform a generic rotation until axis coincide. Using the method of **Euler Angles**, namely based on three composed and finite rotations given in a pre-establish order, and considering the **Tailt-Bryan Convention**, also referred to as **Convention 321**:

   - Deduce the first transformation matrix. [**0.5 pt**]

   [**Estimated time: 8 min; Complete it in the space given below**]

6. State the different velocity regimes according to the Mach number. Explain the particularities of each regime: [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

Test: The test contains 15 questions, each with 3 options. Mark the options you think are correct. There might be none, one, two or three correct options. If you have marked all (in case there is at least one) the correct option/s, you get 0.1 points. If you do not mark all the correct option/s (in case there is at least one), you get 0 points. There might be questions in which none of the options is correct. In such a case you shouldn't mark any to get 0.1 points.

Marked on blue are the correct answers:

1. Regarding the system SACTA: [**0.1 pt**]
   (a) It is a software to assist air traffic controllers.
   (b) It was implemented first in the USA.
   (c) It was developed by spanish companies.

2. Which of the following is being manufactured in Illescas: [**0.1 pt**]
   (a) A350 wing low cover.
   (b) A350 wing up cover.
   (c) A380 section 19.

   In the question that follows related to ISA atmosphere two different answer are considered correct. This is due to the lack of specification in the layer which the question refers to:

3 Which of the following hypotheses hold in the International Standard Atmosphere model: [**0.1 pt**]
   (a) The air is a real gas.
   (b) The variation of temperature with altitude is constant.
   (c) The temperature at 11000 m is 260K.

   Or

   (a) The air is a real gas.
   (b) The variation of temperature with altitude is constant.
   (c) The temperature at 11000 m is 260K.

4 Regarding the different barometric altimeter settings, which of the following statements is true: [**0.1 pt**]

(a) In QNE setting the reference pressure is the pressure at sea level.

(b) In QNH setting the reference pressure is 101325 Pa.

(c) In QFE setting the reference altitude is $h = 0$ m.

5 Consider a stream tube and two sections, namely A and B. If $\rho_A = 2 \cdot \rho_B$, and $S_A = 5S_B$, being ρ_A and ρ_B the density of the fluid in the different sections, and S_A and S_B the area of the sections of the tube, then: [**0.1 pt**]

(a) $V_B = 0.2 \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.

(b) $V_B = 5 \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.

(c) $V_B = 2 \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.

In the question that follows related to Bernouilli equation two different answer are considered correct. This is due to an error on my side: the fact that the solutions are expressed in unities of acceleration while I meant to expressed them in unities of velocity. Thus, no one of the solutions is correct, but I also acept the solution $V_A = 2k$.

6 Consider Bernouilli equation neglecting gravity effects. Considering also that along a stream line there is an stagnation point at which the pressure (p_T) is kwon: $p_T = 2 \cdot k^2$ Pa. We want to calculate the velocity of the fluid at another point A on the stream line. At that point, the pressure $p_A = k^2$ Pa and the density $\rho_A = 0.5$ kg/m^3 are known. Then, the velocity in A is:

(a) $V_A = 2 \cdot k$ m/s^2.

(b) $V_A = 4 \cdot k$ m/s^2.

(c) $V_A = 0.5 \cdot k$ m/s^2..

Or

(a) $V_A = 2 \cdot k$ m/s^2.

(b) $V_A = 4 \cdot k$ m/s^2.

(c) $V_A = 0.5 \cdot k$ m/s^2..

7 In which of the following principles are the passive high-lift devices based:

(a) Increase the camber.
(b) Control the boundary layer.
(c) Increase the wing-span.

8 The ribs:

(a) Give shape to the wing section and support the skin preventing bucking.
(b) Serve as attachment points for control surfaces, flaps, landing gear and engines.
(c) Serve to achieve the aerodynamic shape.

9 In a typical T-arrangement layout for the instruments: [**0.1 pt**]

(a) The airspeed indicator is located on top-left.
(b) The altimeter is located on top-right.
(c) The heading indicator is located on down-center.

10 The turbine pressure ratio (TPR) can be defined as: [**0.1 pt**]

(a) $TPR = \frac{p_{5t}}{p_{4t}}$.
(b) $TPR = (\frac{T_{5t}}{T_{4t}})^{\frac{\gamma-1}{\gamma}}$.
(c) $TPR = \frac{p_{4t}}{p_{3t}}$.

11 Regarding a PID controller, mark those sentences that are true among the following:

(a) The proportional gain, K_p, increases the overshoot of the response.
(b) The derivative gain, K_d, decreases the overshoot of the response.
(c) The integral gain, K_i, increases the rise time.

12 In the terminal area of an airport designates a set of infrastructures among which we can count:

(a) Intermodal connections.
(b) Auxiliary aeronautical buildings such control tower, fire extinction building, electrical building.
(c) Passenger processing building.

13 Regarding the market of aircrafts for commercial air transportation, mark those sentences that are true among the following:

(a) The A320 is a long-range jet.
(b) The A320neo is a series of enhanced versions of the A320 family with a new engine option that is expected to result in 15% of fuel savings.
(c) According to Airbus 2012 average price list, the cost of the A320 family is above 100 Mio USD..

14 Regarding dead reckoning, mark those sentences that are true among the following::

(a) The desired track angle is the angle between the north and the straight line joining two consecutive waypoints.
(b) The track angle is the angle between the north and the absolute velocity of the aircraft.
(c) The cross track error is the distance between the position of the aircraft and the current track.

15 Mark those sentences that are true among the following:

(a) An NDB uses radiogoniomtry, the observable is the distance, and the situation surface is a sphere.
(b) A VOR uses spatial modulation, the observable is the course, and the situation surface is a plane.
(c) A DME uses radiotelemetry, the observable is the course, and the situation surface is a plane.

2.2 Problems

Problems: 1 hour and 50 minutes; 4 points out of 10.

Problem 1 [1 pt] [Estimated time: 35 min]

After the launch of a spatial probe into a planetary atmosphere, data about the temperature of the atmosphere have been collected such its variation with altitude (h) can be approximated as follows:

$$T = \frac{A}{1 + e^{\frac{h}{B}}}, \tag{2.11}$$

where A and B are constants to be determined.

Assuming the gas behaves as a perfect gas and the atmosphere is at rest, using the following data:

- Temperature at $h = 1000$, $T_{1000} = 250$ K;
- $p_0 = 100000 \frac{N}{m^2}$;
- $\rho_0 = 1 \frac{Kg}{m^3}$;
- $T_0 = 300$ K;
- $g = 10 \frac{m}{s^2}$.

determine:

1. The values of A and B, including their unities. [**0.2 pt**]
2. Variation law of density and pressure with altitude, respectively $\rho(h)$ and $p(h)$ (Do not substitute any value). [**0.5 pt**]
3. The value of density and pressure at $h = 1000$ m. [**0.15 pt**]
4. Molecular mass of the gas. Notice that the universal constant for a perfect gas is $R = 8.3144$ J/Mol K. [**0.15 pt**]

Solution to Problem 1 [1 pt].

We assume the following hypotheses:

a) The gas is a perfect gas.

b) It fulfill the fluidostatic equation.

Based on hypothesis a):

$$P = \rho R T. \tag{2.12}$$

Based on hypothesis b):

$$dP = -\rho g dh. \tag{2.13}$$

Based on the data given in the statement, and using Equation (2.12):

$$R = \frac{P_0}{\rho_0 T_0} = 333.3 \frac{J}{(Kg \cdot K)}. \tag{2.14}$$

1. The values of A and B:

Using the given temperature at an altitude $h = 0$ ($T_0 = 300$ K), and Equation (2.11):

$$300 = \frac{A}{1 + e^0} = \frac{A}{2} \rightarrow A = 600\ K \quad [\mathbf{0.1pt}]. \tag{2.15}$$

Using the given temperature at an altitude $h = 1000$ ($T_{1000} = 250$ K), and Equation (2.11):

$$250 = \frac{A}{1 + e^{\frac{1000}{B}}} = \frac{600}{1 + e^{\frac{1000}{B}}} \rightarrow B = 2972\ m \quad [\mathbf{0.1pt}]. \tag{2.16}$$

2. Variation law of density and pressure with altitude:

Using Equation (2.12) and Equation (2.13):

$$dP = -\frac{P}{RT} g dh \quad [\mathbf{0.1pt}]. \tag{2.17}$$

Integrating the differential Equation (2.17) between $P(h = 0)$ and P; $h = 0$ and h:

$$\int_{P_0}^{P} \frac{dP}{P} = \int_{h=0}^{h} -\frac{g}{RT} dh. \tag{2.18}$$

Introducing Equation (2.11) in Equation (2.18):

$$\int_{P_0}^{P} \frac{dP}{P} = \int_{h=0}^{h} -\frac{g(1+e^{\frac{h}{B}})}{RA} dh \quad [\mathbf{0.1pt}]. \tag{2.19}$$

Integrating Equation (2.19):

$$Ln\frac{P}{P_0} = -\frac{g}{RA}(h + Be^{\frac{h}{B}} - B) \rightarrow P = P_0 e^{-\frac{g}{RA}(h+Be^{\frac{h}{B}}-B)}. \quad [\mathbf{0.2pt}]. \tag{2.20}$$

Using Equation (2.12), Equation (2.11), and Equation (2.20):

$$\rho = \frac{P}{RT} = \frac{P_0 e^{-\frac{g}{RA}(h+Be^{\frac{h}{B}}-B)}}{R\frac{A}{1+e^{\frac{h}{B}}}}. \quad [\mathbf{0.1pt}]. \tag{2.21}$$

3. Pressure and density at an altitude of 1000 m:

Using Equation (2.20) and Equation (2.21), the given data for P_0 and g, and the values obtained for R, A and B:

- $\rho(h = 1000) = 1.0756\ \frac{kg}{m^3}$. [**0.075 pt**].
- $P(h = 1000) = 89632.5\ Pa$. [**0.075 pt**].

4. Molecular mass of the gas:

As it has been stated in the statement, the universal constant for a perfect gas is $R_{univ} = 8.3144\ \frac{J}{Mol \cdot K}$. The constant for the gas subject to study in this problem is: $R = 333.3\ \frac{J}{Kg \cdot K}$.

Therefore, given that the molecular gas of the gas, M, can be calculated as:

$$M = \frac{R_{univ}}{R} = 0.024\ \frac{Kg}{Mol}. \quad [\mathbf{0.15pt}]. \tag{2.22}$$

Problem 2 [1.5 pt] [Estimated time: 40 min]

We want to estimate the take-off distance of an Airbus A-320 taking off at Madrid-Barajas airport. Such aircraft mounts two turbojets, whose thrust can be estimated as: $T = T_0(1 - k \cdot V^2)$, where T is the thrust, T_0 is the nominal thrust, k is a constant and V is the true airspeed.

Considering that:

- $g \cdot \left(\frac{T_0}{m \cdot g} - \mu_r\right) = 1.31725 \ \frac{m}{s^2}$;
- $\frac{\rho S (C_D - \mu_r C_L) + 2 \cdot T_0 \cdot k}{2 \cdot m} = 3.69 \cdot 10^{-5} \frac{m}{s^2}$;
- The velocity of take off is $V_{TO} = 70$ m/s;

where g is the force due to gravity, m is the mass of the aircraft, μ_r is the friction coefficient, ρ is the density of air, S is the wet surface area of the aircraft, C_D is the coefficient of drag and C_L is the coefficient of lift[2].

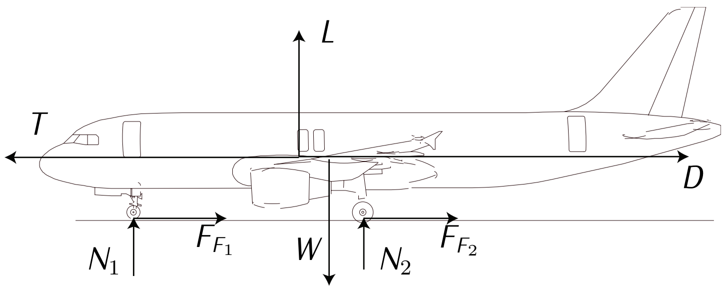


Figure 2.2: Forces during taking off.

Calculate:

1. Take-off distance. [**1.5 pt**]

[2] All this variables can be considered constant during take off

Solution to Problem 2 [1.5 pt].

We apply the 2nd Newton's Law:

$$\sum F_z = 0 \tag{2.23}$$
$$\sum F_x = m\dot{V}. \tag{2.24}$$

Regarding Equation (2.23), Notice that while rolling on the ground, the aircraft is assumed to be under equilibrium along the vertical axis.

Looking at Figure 2.2, Equations (2.23-2.24) become:

$$L + N - mg = 0 \quad [\mathbf{0.15pt}]. \tag{2.25}$$
$$T - D - F_F = m\dot{V}, \quad [\mathbf{0.15pt}]. \tag{2.26}$$

being L the lift, N the normal force, mg the weight; T the trust, D the drag and F_F the total friction force.

It is well known that:

$$L = C_L\frac{1}{2}\rho SV^2, \tag{2.27}$$
$$D = C_D\frac{1}{2}\rho SV^2. \tag{2.28}$$

It is also well known that:

$$F_F = \mu_r N. \tag{2.29}$$

Equation (2.25) states that: $N = mg - L$. Therefore:

$$F_F = \mu_r(mg - L). \quad [\mathbf{0.15pt}]. \tag{2.30}$$

Given that $T = T_0(1 - kV^2)$, with Equation (2.30) and Equations (3.13-3.14), Equation (2.26) becomes:

$$(\frac{T_0}{m} - \mu_r g) + \frac{(\rho S(\mu_r C_L - C_D) - 2T_0 k)}{2m}V^2 = \dot{V}. \quad [\mathbf{0.3pt}]. \tag{2.31}$$

Now, we have to integrate Equation (2.31).

In order to do so, we know, as it was stated in the statement, that: $T_0, m, \mu_r, g.\rho, S, C_L, C_D$ *and* k can be considered constant along the take off phase.

We have that:

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt}, \quad [\mathbf{0.1pt}]. \tag{2.32}$$

and knowing that $\frac{dx}{dt} = V$, Equation (2.31) becomes:

$$\frac{(\frac{T_0}{m} - \mu_r g) + \frac{(\rho S(\mu_r C_L - C_D) - 2T_0 k)}{2m} V^2}{V} = \frac{dV}{dx}. \quad [\mathbf{0.1pt}]. \tag{2.33}$$

In order to simplify Equation (2.34):

- $(\frac{T_0}{m} - \mu_r g) = g(\frac{T_0}{mg} - \mu_r) = A$
- $\frac{(\rho S(\mu_r C_L - C_D) - 2T_0 k)}{2m} = B$

We proceed on integrating Equation (2.34) between $x = 0$ and x_{T0} (the distance of take off); $V = 0$ (Assuming the maneuver starts with the aircraft at rest) and the velocity of take off that was given in the statement: $V_{TO} = 70$ m/s. It holds that:

$$\int_0^{x_{TO}} dx = \int_0^{V_{TO}} \frac{VdV}{A + BV^2}. \tag{2.34}$$

Integrating:

$$\Big[x\Big]_0^{x_{TO}} = \Big[\frac{1}{2B} Ln(A + BV^2)\Big]_0^{V_{TO}} \quad [\mathbf{0.2pt}] \tag{2.35}$$

Substituting the upper and lower limits:

$$x_{TO} = \frac{1}{2B} Ln(1 + \frac{B}{A} V_{TO}^2) \quad [\mathbf{0.2pt}] \tag{2.36}$$

Substituting the data given in the statement:

- $A = 1.31725\ \frac{m}{s^2}$
- $B = -3.69 \cdot 10^{-5}\ \frac{m}{s^2}$
- $V_{TO} = 70\ m/s$

The distance to take off is $x_{TO} = 2000\ m$ [**0.15 pt**]

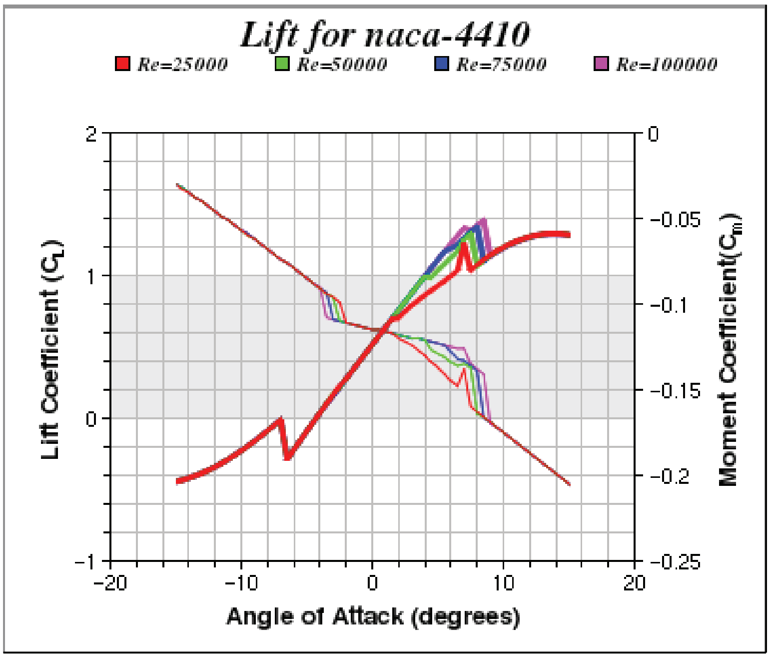
Problem 3 [1.5 pt] [Estimated time: 35 min]

We want to know the aerodynamic characteristics of a NACA-4410 airfoil for a Reynolds number Re=100000. Experimental results gave the characteristic curves shown in Figure 2.3.

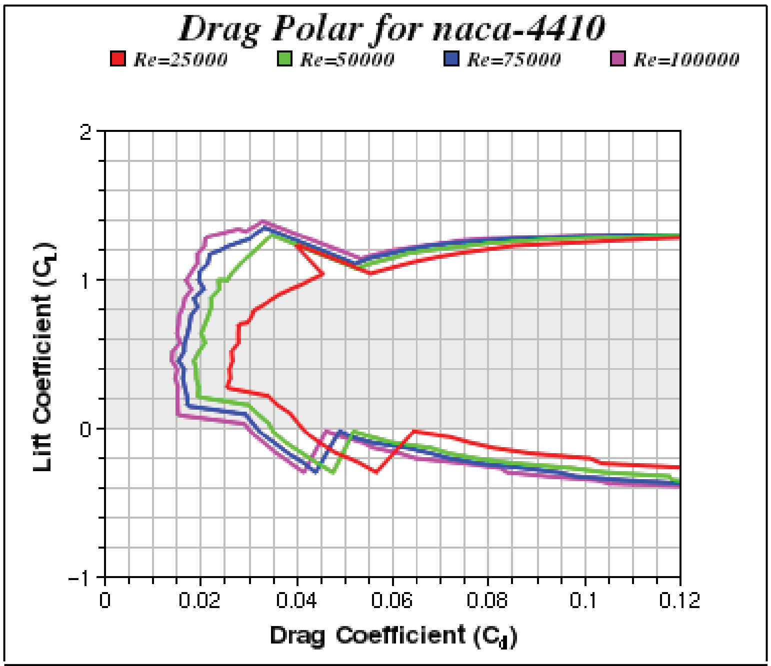
Looking at Figure

Calculate:

1. The expression of the lift curve in the linear range in the form: $c_l = c_{l0} + c_{l\alpha}\alpha$. [**0.2 pt**]
2. The expression of the parabolic polar of the airfoil in the form: $c_d = c_{d0} + bc_l + kc_l^2$. [**0.4 pt**]
3. The angle of attack and the coefficient of lift corresponding to the minimum coefficient of drag. [**0.2 pt**]
4. The angle of attack, the coefficient of lift and the coefficient of drag corresponding to the maximum aerodynamic efficiency. [**0.4 pt**]
5. The values of the aerodynamic forces per unity of longitude that the model with chord $c = 2$ m would produce in the wind tunnel experiments with an angle of attack of $\alpha = 3°$and incident current with Mach number M=0.3. Consider ISA conditions at an altitude of $h = 1000$ m. [**0.3 pt**]



(a) $c_l(\alpha)$ and $c_m(\alpha)$



(b) $c_l(c_d)$

Figure 2.3: Characteristic curves on a NACA 4410 airfoil

Solution to Problem 3 [1.5 pt].

We want to approximate the experimental data given in Figure (2.3), respectively to a straight line and a parabolic curve. Therefore, to univocally define such curves, we must choose:

- Two pair of points (c_l, α) of the $c_l(\alpha)$ curve in Figure (2.3.a).
- Three pair of points (c_l, c_d) of the $c_l(c_d)$ curve in Figure (2.3.b).

According to Figure 2.3 for Re=100000 we choose (any other combination properly chosen must work):

c_l	α
0.5	0°
1	4°

Table 2.1: Data obtained from Figure (2.3.a).

c_l	c_d
0	0.03
1	0.0175
1.4	0.0325

Table 2.2: Data obtained from Figure (2.3.b).

1. The expression of the lift curve in the linear range in the form: $c_l = c_{l0} + c_{l\alpha}\alpha$;

With the data in Table 2.1:

$$c_{l0} = 0.5 \quad [\mathbf{0.1pt}] \tag{2.37}$$
$$c_{l\alpha} = 7.16 \cdot 1/rad \quad [\mathbf{0.1pt}] \tag{2.38}$$

The required curve yields then:

$$c_l = 0.5 + 7.16\alpha \quad [\alpha \; in \; rad] \tag{2.39}$$

2. The expression of the parabolic polar of the airfoil in the form: $c_d = c_{d0} + bc_l + kc_l^2$;

With the data in Table 2.2 we have a system of three equations with three unkonws that must be solve. It yields:

$$c_{d0} = 0.03 \quad [\mathbf{0.1pt}] \tag{2.40}$$
$$b = -0.048 \quad [\mathbf{0.1pt}] \tag{2.41}$$
$$k = 0.0357 \quad [\mathbf{0.1pt}] \tag{2.42}$$

The expression of the parabolic polar yields:

$$c_d = 0.03 - 0.048c_l + 0.0357c_l^2 \quad [\mathbf{0.1pt}] \tag{2.43}$$

3. The angle of attack and the coefficient of lift corresponding to the minimum coefficient of drag.

In order to do so, we seek the minimum of the parabolic curve:

$$\frac{dc_l}{dc_d} = 0 = b + 2 \cdot kc_l \quad [\mathbf{0.1pt}] \tag{2.44}$$

Substituting in Equation (3.5):

$$\frac{dc_l}{dc_d} = 0 = -0.048 + 2 \cdot 0.0357c_l \rightarrow (c_l)_{c_{d_{min}}} = 0.672 \quad [\mathbf{0.05pt}] \tag{2.45}$$

Substituting $(c_l)_{c_{d_{min}}}$ in Equation (2.39), we obtain:

$$(\alpha)_{c_{d_{min}}} = 0.024\ rad\ (1.378^\circ) \quad [\mathbf{0.05pt}]$$

4. The angle of attack, the coefficient of lift and the coefficient of drag corresponding to the maximum aerodynamic efficiency.

The aerodynamic efficiency is defined as:

$$E = \frac{l}{d} = \frac{c_l}{c_d} \tag{2.46}$$

Substituting the parabolic polar curve in Equation (2.46), we obtain:

$$E = \frac{c_l}{c_{d0} + bc_l + kc_l^2} \tag{2.47}$$

In order to seek the values corresponding to the maximum aerodynamic efficiency, one must derivate and make it equal to zero, that is:

$$\frac{dE}{dc_l} = 0 = \frac{c_{d0} - kc_l^2}{(c_{d0} + bc_l + kc_l^2)^2} \rightarrow (c_l)_{E_{max}} = \sqrt{\frac{c_{d0}}{k}}. \quad [\mathbf{0.2pt}] \tag{2.48}$$

Substituting according to the values previously obtained ($c_{d0} = 0.03$, $k = 0.0357$): $(c_l)_{E_{max}} = 0.91$ **[0.05 pt]** Substituting in Equation (2.39) and Equation (3.5), we obtain:

- $(\alpha)_{E_{max}} = 0.058$ rad (3.33°) **[0.075 pt]**
- $(c_d)_{E_{max}} = 0.01588$ **[0.075 pt]**

5. The values of the aerodynamic forces per unity of longitude that the model with chord $c = 2$ m would produce in the wind tunnel experiments with an angle of attack of $\alpha = 3°$and incident current with Mach number M=0.3.

According to ISA:

- $\rho(h = 1000) = 0.907$ Kg/m^3
- $a(h = 1000) = \sqrt{\gamma_{air} R(T_0 - \lambda h)} = 336.4$ m/s

where a corresponds to the speed of sound, $\gamma_{air} = 1.4$, $R = 287$ J/KgK, $T_0 = 288.15$ k and $\lambda = 6.5 \cdot 10^{-3}$.

Since the experiment is intended to be at M=0.3:

$$V = M \cdot a = 100.92\ m/s \quad [\mathbf{0.1pt}] \tag{2.49}$$

Since the experiment is intended to be at $\alpha = 3°$, using Equation (2.39) and Equation (3.5):

$$c_l = 0.87 \quad [\mathbf{0.05pt}] \tag{2.50}$$

$$c_d = 0.01526 \quad [\mathbf{0.05pt}] \tag{2.51}$$

Finally:

$$l = c_l \frac{1}{2} \rho c V^2 = 8036.77\ N/m \quad [\mathbf{0.05pt}] \tag{2.52}$$

$$d = c_d \frac{1}{2} \rho c V^2 = 140.96\ N/m \quad [\mathbf{0.05pt}] \tag{2.53}$$

3

Recovery exam

Recovery Exam, 28th June, 2012

3.1 Theory

Theory: 1 hour and 50 minutes; 6 points out of 10.

Exposition questions: **Expose and develop the posed questions. Use additional paper to assess the questions. In order to have an quantitative measure, the estimated times and spaces are given.**

1. International Standard Atmosphere:

   (a) Deduce the equations of the International Standard Atmosphere (ISA) in the troposphere ($0 \leq h \leq 11000$ [m]): [**0.4 pt**]

   - Expose the considered hypotheses.
   - Deduce the equations that relate pressure and density with altitude.

   (b) Based on the previously calculated equations, deduce the barometric altitude equation[1]. [**0.2 pt**]

   (c) Define the different altimeter settings, namely QNE, QNH and QFE, stating the reference values for altitude and pressure. [**0.15 pt**]

   [**Estimated time: 20 min; Estimated space: 1 paper side**]

[1]Notice that the barometric altitude equation is that relating the altitude with a reference altitude, a measured pressure and a reference pressure.

2. Longitudinal balancing:

   (a) Explain in what does the longitudinal balancing problem consist. [**0.2 pt**]

   (b) Based on Figure 3.1:

      i. State what the terms that appear represent and its sign criteria. [**0.15 pt**]

      ii. Deduce the equations of the longitudinal equilibrium. [**0.2 pt**]

   (c) How does a defection of the elevator affects the equilibrium. [**0.2 pt**]

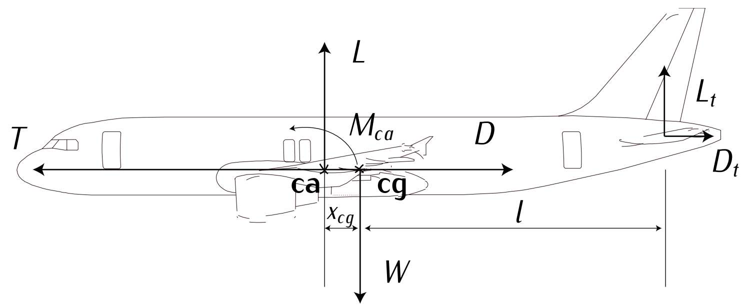


Figure 3.1: Longitudinal equilibrium.

[**Estimated time: 20 min; Estimated space: 1 paper side**]

3. Chicago's Convention:

   (a) Explain what the Chicago's Convention is. [**0.25 pt**]

   (b) Why it is considered as a fundamental milestone in the development of the international air transportation system. [**0.5 pt**]

[**Estimated time: 20 min; Estimated space: 1 paper side**]

4. The CNS/ATM concept:

   (a) Expose the CNS/ATM concept within the air navigation system. [**0.75 pt**]

[**Estimated time: 20 min; Estimated space: 1 paper side**]

Short questions: **Brief statement of the posed questions. It is compulsory to use the space given bellow the statements to assess the questions.**

5. Define what a rotorcraft is, stating the different types and pointing out some of their characteristics. [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

6. State briefly what is a composite material, what are its main components and what are its structural characteristics. [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

7. Explain what the effective angle of attack is. Use a sketch to clarify it. [**0.5 pt**]

   [**Estimated time: 5 min; Complete it in the space given below**]

8. Define briefly what a inertial navigation system is. [**0.5 pt**]
   [**Estimated time: 5 min; Complete it in the space given below**]

9. Attending at Figure 3.2, where a runway is orientated with an angle of 22 degrees with respect to a given reference, obtain the designators of both runway heads. [**0.5 pt**]

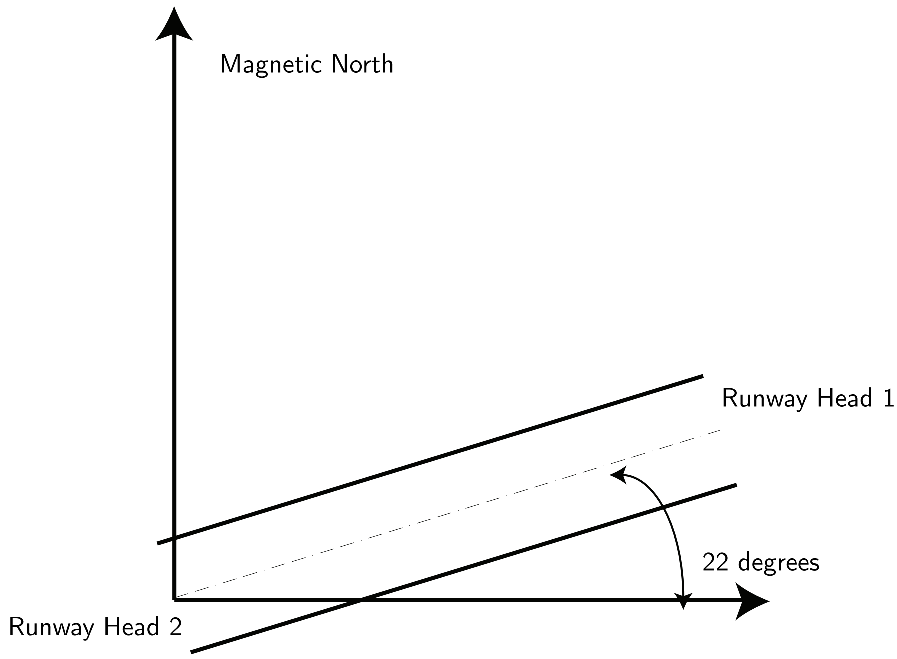


Figure 3.2: Runway Designators.

[**Estimated time: 5 min; Complete it in the space given below**]

Test: **The test contains 5 questions, each with 3 options. Mark the options you think are correct. There might be none, one, two or three correct options. If you have marked all (in case there is at least one) the correct option/s, you get 0.1 points. If you do not mark all the correct option/s (in case there is at least one), you get 0 points. There might be questions in which none of the options is correct. In such a case you shouldn't mark any to get 0.1 points.**

Marked on blue are the correct answers:

1. Consider a stream tube and two sections, namely A and B. If the flow is considered incompressible, and $S_A = 4S_B$, being ρ_A and ρ_B the density of the fluid in the different sections, and S_A and S_B the area of the sections of the tube, then: [**0.1 pt**]
   (a) $V_B = 0.25 \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.
   (b) $V_B = 4 \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.
   (c) $V_B = 0.25 \cdot \frac{\rho_A}{\rho_B} \cdot V_A$, being V_A and V_B the velocity of the fluid in Sections A and B.

2. Consider Bernouilli equation considering gravity effects. Considering also that along a stream line there is an stagnation point at which the pressure (p_T) is kwon: $p_T = 2 \cdot k^2$ Pa, and the altitude (z_T) is also known: $z_T = \frac{k^2}{\rho_T g_T}$ m. We want to calculate the velocity of the fluid at another point A on the stream line. At that point, the pressure $p_A = k^2$ Pa, the altitude is $z_A = 0.1$ m, the gravity is $g_A = 10$ m/s^2 and the density $\rho_A = k^2$ kg/m^3 are known. Then, the velocity in A is: [**0.1 pt**]
   (a) $V_A = 2 \cdot k$ m/s.
   (b) $V_A = \sqrt{2}$ m/s.
   (c) $V_A = \sqrt{2} \cdot k$ m/s.

3. Which of the following statements are true: [**0.1 pt**]
   (a) The Mach number has the unities of a velocity (m/s).
   (b) The coefficient of pressures is dimensionless.
   (c) A moment has the unities of an energy (Joule).

4. Consider that the Block Time can be linearly approximated as $BT = \alpha + \beta R$, being R the range of the aircraft, α and β constants. If the operative cost of a company can be calculated as $c = c_f + c_v BT$, being c_f the fixed costs and c_v the variable costs, the cost per kilometer can be expressed as: [**0.1 pt**]

   (a) $c_v + \frac{c_f}{\alpha+\beta R}$.

   (b) $\frac{c_f+c_v\alpha}{R} + \beta c_v$.

   (c) $c_f + c_v(\alpha + \beta R)$.

5. What of the following can be considered as main goals of SESAR: [**0.1 pt**]

   (a) Increase the capacity of the air space by a factor of 3.

   (b) Increase the safety of the air navigation system by a factor of 10.

   (c) 10% reduction of aircraft's environmental footprint.

3.2 Problems

Problems: 1 hour and 30 minutes; 4 points out of 10.

Problem 1 [2 pt] [Estimated time: 40 min]

We want to analyze the aerodynamic performances of a trapezoidal wing with a plant-form as in Figure 3.3 and an efficiency factor of the wing of $e = 0.96$. Moreover, we will employ a NACA 4415 airfoil with the following characteristics:

- $c_l = 0.2 + 5.92\alpha$.
- $c_d = 6.4 \cdot 10^{-3} - 1.2 \cdot 10^{-3} c_l + 3.5 \cdot 10^{-3} c_l^2$.

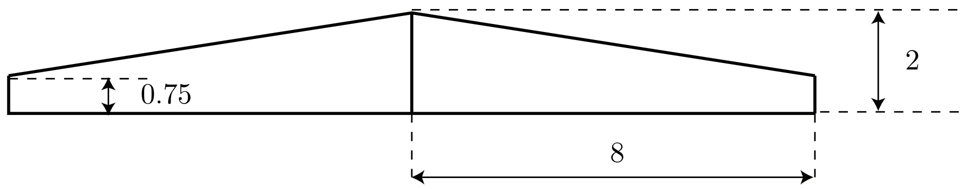


Figure 3.3: Plant-form of the wing. Dimensions in meters.

Calculate:

1. The following parameters of the wing[2]: chord at the root; chord at the tip; mean chord; wing-span; wet surface; enlargement. [**0.2 pt**]
2. The lift curve of the wing in the linear range. [**0.4 pt**]
3. The polar of the wing assuming that it can be calculated as $C_D = C_{D_0} + C_{D_i} C_L^2$. [**0.2 pt**]
4. Calculate the optimal coefficient of lift, $C_{L_{opt}}$ for the wing. Compare it with the airfoils's one. [**0.4 pt**]
5. Calculate the optimal coefficient of drag, $C_{D_{opt}}$ for the wing. Compare it with the airfoils's one. [**0.2 pt**]
6. Maximum aerodynamic efficiency E_{max} for the wing. Compare it with the airfoils's one. [**0.2 pt**]
7. Discuss the differences observed in $C_{L_{opt}}$, $C_{D_{opt}}$ and E_{max} between the wing and the airfoil. [**0.4 pt**]

[2] Based on the given data in Figure 3.3.

Solution to Problem 1 [2 pt].

1. Chord at the root; chord at the tip; mean chord; wing-span; wet surface; enlargement:

According to Figure 3.3:

- The wing-span, b, is $b = 16$ m.
- The chord at the tip, c_t, is $c_t = 0.75$ m.
- The chord at the root, c_r, is $c_r = 2$ m.

We can also calculate the wet surface of the wing calculating twice the area of a trapezoid as follows:

$$S_w = 2\left(\frac{(c_r + c_t)}{2}\frac{b}{2}\right) = 22\ m^2 \quad [\mathbf{0.1pt}].$$

The mean chord, $\bar{c}$, can be calculated as $\bar{c} = \frac{S_w}{b} = 1.375$ m **[0.05 pt]**.; and the enlargement, A, as $A = \frac{b}{\bar{c}} = 11.63$ **[0.05 pt]**.

2. Wing's lift curve:

The lift curve of a wing can be expressed as follows:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \quad [\mathbf{0.05pt}], \tag{3.1}$$

and the slope of the wing's lift curve can be expressed related to the slope of the airfoil's lift curve as:

$$C_{L_\alpha} = \frac{C_{l_\alpha}}{1 + \frac{C_{l_\alpha}}{\pi A}}e = 4.89 \cdot\ 1/rad \quad [\mathbf{0.05pt}].$$

In order to calculate the independent term of the wing's lift curve, we must consider the fact that the zero-lift angle of attack of the wing coincides with the zero-lift angle of attack of the airfoil, that is:

$$\alpha(L = 0) = \alpha(l = 0) \quad [\mathbf{0.1pt}]. \tag{3.2}$$

First, notice that the lift curve of an airfoil can be expressed as follows

$$C_l = C_{l_0} + C_{l_\alpha}\alpha. \tag{3.3}$$

Therefore, with Equation (3.1) and Equation (3.3) in Equation (??), we have that:

$$C_{L_0} = C_{l_0}\frac{C_{L_\alpha}}{C_{l_\alpha}} = 0.165 \quad [\mathbf{0.1pt}].$$

The required curve yields then:

$$C_L = 0.165 + 4.89\alpha \ \ [\alpha\ in\ rad] \quad [\mathbf{0.1pt}].$$

3. The expression of the parabolic polar of the wing;

Notice first that the statement of the problem indicates that the polar should be in the following form:

$$C_D = C_{D0} + C_{D_i} C_L^2; \tag{3.4}$$

For the calculation of the parabolic drag of the wing we can consider the parasite term approximately equal to the parasite term of the airfoil, that is, $C_{D_0} = C_{d_0}$ **[0.05 pt]**.

The induced coefficient of drag can be calculated as follows:

$$C_{D_i} = \frac{1}{\pi A e} = 0.028 \quad [\mathbf{0.05pt}].$$

The expression of the parabolic polar yields then:

$$C_D = 0.0064 + 0.028 C_L^2 \quad [\mathbf{0.1pt}]. \tag{3.5}$$

4. The optimal coefficient of lift, $C_{L_{opt}}$, for the wing. Compare it with the airfoils's one.

The optimal coefficient of lift is that making the aerodynamic efficiency maximum. The aerodynamic efficiency is defined as:

$$E = \frac{L}{D} = \frac{C_L}{C_D} \tag{3.6}$$

Substituting the parabolic polar curve given in Equation (3.15) in Equation (3.6), we obtain:

$$E = \frac{C_L}{C_{D_0} + C_{D_i} C_L^2} \tag{3.7}$$

In order to seek the values corresponding to the maximum aerodynamic efficiency, one must derivate and make it equal to zero, that is:

$$\frac{dE}{dC_L} = 0 = \frac{C_{D_0} - C_{D_i} C_L^2}{(C_{D_0} + C_{D_i} C_L^2)^2} \rightarrow (C_L)_{E_{max}} = C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{C_{D_i}}}. \quad [\mathbf{0.15pt}] \tag{3.8}$$

For the case of an airfoil, the aerodynamic efficiency is defined as:

$$E = \frac{l}{d} = \frac{c_l}{c_d} \tag{3.9}$$

Substituting the parabolic polar curve given in the statement in the form $c_{d_0} + b c_l + k c_l^2$ in Equation (3.9), we obtain:

$$E = \frac{c_l}{c_{d_0} + b c_l + k c_l^2} \tag{3.10}$$

In order to seek the values corresponding to the maximum aerodynamic efficiency, one must derivate and make it equal to zero, that is:

$$\frac{dE}{dC_l} = 0 = \frac{c_{d_0} - kc_l^2}{(c_{d0} + bc_l + kc_l^2)^2} \rightarrow (c_l)_{E_{max}} = (c_l)_{opt} = \sqrt{\frac{c_{d_0}}{k}}. \quad [\mathbf{0.15pt}] \tag{3.11}$$

According to the values previously obtained ($C_{D_0} = 0.0064$ and $C_{D_i} = 0.028$) and the values given in the statement for the airfoil's polar ($c_{d_0} = 0.0064$, $k = 0.0035$), substituting them respectively in Equation (3.8) and Equation (3.11), we obtain :

- $(C_L)_{opt} = 0.478$ **[0.05 pt]**
- $(c_l)_{opt} = 1.35$ **[0.05 pt]**

5. The optimal coefficient of drag, $C_{D_{opt}}$ for the wing. Compare it with the airfoils's one.

Once the optimal coefficient of lift has been obtained for both airfoil and wing, simply by substituting their values into both parabolic curves given respectively in the statement and in Equation 3.5, we obtain:

$$C_{D_{opt}} = 0.0064 + 0.028 C_{L_{opt}}^2 = 0.01279. \quad [\mathbf{0.1pt}].$$
$$c_{d_{opt}} = 6.4 \cdot 10^{-3} - 1.2 \cdot 10^{-3} c_{l_{opt}} + 3.5 \cdot 10^{-3} c_{l_{opt}}^2 = 0.01115. \quad [\mathbf{0.1pt}].$$

6. Maximum aerodynamic efficiency E_{max} for the wing. Compare it with the airfoils's one.

The maximum aerodynamic efficiency can be obtained as:

$$E_{max_{wing}} = \frac{C_{L_{opt}}}{C_{D_{opt}}} = 37. \quad [\mathbf{0.1pt}].$$
$$E_{max_{airfoil}} = \frac{c_{l_{opt}}}{c_{d_{opt}}} = 121. \quad [\mathbf{0.1pt}].$$

7. Discuss the differences observed in $C_{L_{opt}}$, $C_{D_{opt}}$ and E_{max} between the wing and the airfoil. **[0.4 pt]**

According to the results it is straightforward to see that meanwhile the optimum coefficient of drag is similar for both airfoil and wing, the optimum coefficient of lift is approximately three times lower that the airfoil's one. Obviously this results in an approximately three time lower efficiency for the wing when compared to the airfoil's one.

What does it mean? A three dimensional aircraft made of 2D airfoils generates much more drag than the 2D airfoil in order to achieve a required lift. Therefore, we can not simply extrapolate the analysis of an airfoil to the wing.

Such loss of efficiency is due to the so-called induced drag by lift. The explanation behind this behavior is due to the difference of pressures between extrados and intrados. In particular, in the region close to the marginal border, there is an air current surrounding the marginal border which passes from the intrados, where the pressure is higher, to extrados, where the pressure is lower, giving rise to two vortexes, one in each border rotating clockwise and counterclockwise. This phenomena produces downstream a whirlwind trail.

The presence of this trail modifies the fluid filed and, in particular, modifies the velocity each wing airfoil "sees". In addition to the freestream velocity u_∞, a vertical induced velocity u_i must be added (See Figure 3.4). The closer to the marginal border, the higher the induced velocity is. Therefore, the effective angle of attack of the airfoil is lower that the geometric angle, which explains both the reduction in the coefficient of lift (with respect to the bi-dimensional coefficient) and the appearance of and induced drag (See Figure 3.5).

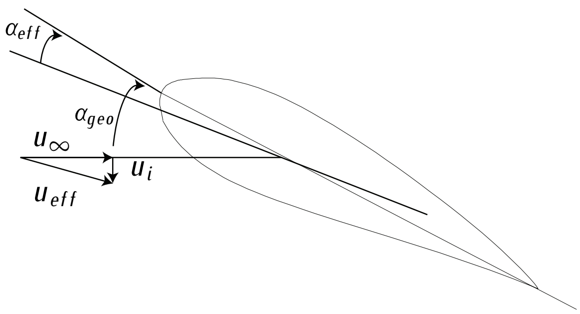


Figure 3.4: Effective angle of attack

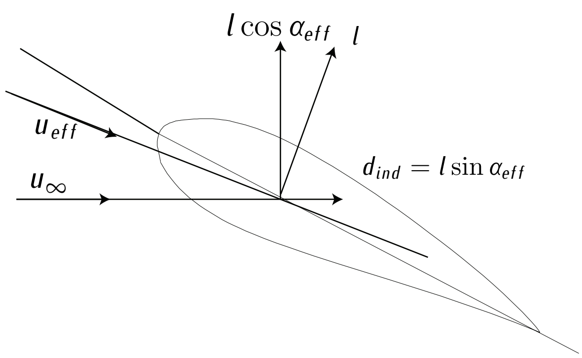


Figure 3.5: Induced drag

Problem 2 [2 pt] [Estimated time: 50 min]

An aircraft has the following characteristics:

- $S_w = 130\ m^2$.
- $b = 40\ m$.
- $m = 70000$ kg.
- $T_{max,av}(h = 0) = 120000$ N (Maximum available thrust at sea level).
- $C_{D_0} = 0.02$.
- Oswald coefficient (wing efficiency coefficient): $e = 0.9$.
- $C_{L_{max}} = 1.5$.

We can consider that the maximum thrust only varies with altitude as follows: $T_{max,av}(h) = T_{max,av}(h = 0)\frac{\rho}{\rho_0}$. Consider standard atmosphere ISA and constant gravity $g = 9.8\ m/s^2$. Determine:

1. The required thrust to fly at an altitude of $h = 11000$ m with $M_\infty = 0.7$ in horizontal, steady, straight flight. [**0.3 pt**]
2. The maximum velocity due to propulsive limitations of the aircraft and the corresponding Mach number in horizontal, steady, straight flight at $h = 11000$. [**0.4 pt**]
3. The minimum velocity due to aerodynamic limitations (stall speed) in horizontal, steady, straight flight at an altitude of $h = 11000$. [**0.15 pt**]
4. The theoretical ceiling (maximum altitude) in horizontal, steady, straight flight. [**0.5 pt**]

We want to perform a horizontal turn at an altitude of $h = 11000$ with a load factor $n = 2$, and with the velocity corresponding to the maximum aerodynamic efficiency in horizontal, steady, straight flight. Determine:

5. The required bank angle. [**0.3 pt**]
6. The radius of turn. [**0.1 pt**]
7. The required thrust. Can the aircraft perform the complete turn? [**0.1 pt**]

We want to perform a horizontal turn with the same load factor and the same radius as in the previous case, but at an altitude corresponding to the theoretical ceiling of the aircraft.

8. Can the aircraft perform such turn? [**0.15 pt**]

Solution to Problem 2 [2 pt].

Besides the data given in the statement, the following data have been used:

- R= 287 J/(kgK)
- $\gamma_{air} = 1.4$
- $\alpha_T = 6.5 \cdot 10^{-3}$ K/m
- $\rho_0 = 1.225\ kg/m^3$
- $T_0 = 288.15$ K
- ISA: $\rho = \rho_0 (1 - \frac{\alpha_T h}{T_0})^{\frac{gR}{\alpha_T} - 1}$

1. Required Thrust to fly a horizontal, steady, straight flight under the following flight conditions:

- $h = 11.000m$.
- $M_\infty = 0.7$

According to ISA:

- $\rho(h = 11000) = 0.364$ Kg/m^3 **[0.025 pt]**
- $a(h = 11000) = \sqrt{\gamma_{air} R (T_0 - \alpha_T h)} = 295.04$ m/s **[0.025 pt]**

where a corresponds to the speed of sound.

The system that governs the dynamics of the aircraft is:

$$T = D, \tag{3.12a}$$
$$L = mg, \tag{3.12b}$$

being L the lift, mg the weight; T the trust and D the drag. **[0.05 pt]**

It is well known that:

$$L = C_L \frac{1}{2} \rho S_w V^2, \tag{3.13}$$
$$D = C_D \frac{1}{2} \rho S_w V^2. \tag{3.14}$$

It is also well known that the coefficient of drag can be expressed in a parabolic form as follows:

$$C_D = C_{D_0} + C_{D_i} C_L^2., \tag{3.15}$$

where C_{D_0} is given in the statement and $C_{D_i} = \frac{1}{\pi A e}$. The enlargement, A, can be calculated as $A = \frac{b^2}{S_w} = 12.30$ and therefore: $C_{D_i} = 0.0287$. **[0.05 pt]**

According to Equation (3.12b): $L = 686000$ N. The velocity of flight can be calculated as $V = M_\infty a = 206.5$ m/s. Once these values are obtained , with the values of density and wet surface, and entering in Equation (3.13), we obtain that $C_L = 0.68$. **[0.05 pt]**

With the values of C_L, C_{D_i} and C_{D_0}, entering in Equation (3.15) we obtain that $C_D = 0.0332$.

Looking now at Equation (3.12a) and using Equation (3.14), we can state that the required thrust is as follows:

$$T = C_D \frac{1}{2} \rho S_w V^2.$$

Since all values are known, the required thrust yields:

$$T = 33567\ N. \quad [\mathbf{0.025pt}]$$

Before moving on, we should look wether the required thrust exceeds or not the maximum available thrust at the given altitude. In order to do that, it has been given that the maximum thrust only varies with altitude as follows:

$$T_{max,av}(h) = T_{max,av}(h = 0)\frac{\rho}{\rho_0} \tag{3.16}$$

The maximum available thrust at h=11000 yields:

$$T_{max,av}(h = 11000) = 35657.14\ N. \tag{3.17}$$

Since $T < T_{max,av}$, the flight condition is flyable.

2. The maximum velocity due to propulsive limitations of the aircraft and the corresponding Mach number in horizontal, steady, straight flight at $h = 11000$:

The maximum velocity due to propulsive limitation at the given altitude implies flying at the maximum available thrust that was obtained in Equation (3.17). **[0.05 pt]**

Looking again at Equation (3.12a) and using Equation (3.14), we can state that:

$$T_{max,av} = C_D \frac{1}{2} \rho S_w V^2. \tag{3.18}$$

Using Equation (3.15) and Equation (3.13), and entering in Equation (3.18) we have that:

$$T_{max,av} = \left(C_{D_0} + C_{D_i} \left(\frac{L}{\frac{1}{2}\rho S_w V^2} \right)^2 \right) \frac{1}{2} \rho S_w V^2 . [\mathbf{0.05pt}] \tag{3.19}$$

Multiplying Equation (3.19) by V^2 we obtain a quadratic equation of the form:

$$ax^2 + bx + c = 0. \tag{3.20}$$

where $x = V^2$, $a = \frac{1}{2}\rho S_w C_{D_0}$,$b = -T_{max,av}$ and $c = \frac{C_{D_i} L^2}{\frac{1}{2}\rho S_w}$. **[0.1 pt]**,

Solving the quadratic equation we obtain two different speeds at which the aircraft can fly given the maximum available thrust[3]. Those velocities yield:

$$V_1 = 228\ m/s,$$
$$V_2 = 151\ m/s. \quad [\mathbf{0.2pt}],$$

The maximum corresponds, obviously, to V_1.

3. The minimum velocity due to aerodynamic limitations (stall speed) in horizontal, steady, straight flight at an altitude of $h = 11000$.

The stall speed takes place when the coefficient of lift is maximum **[0.05 pt]**, therefore, using equation Equation (3.13), we have that:

$$V_{stall} = \sqrt{\frac{L}{\frac{1}{2}\rho S_w C_{L_{max}}}} = 139\ m/s \quad [\mathbf{0.1pt}].$$

4. The theoretical ceiling (maximum altitude) in horizontal, steady, straight flight.

In order to obtain the theoretical ceiling of the aircraft the maximum available thrust at that maximum altitude must coincide with the minimum required thrust to fly horizontal, steady, straight flight at that maximum altitude, that is:

$$T_{max,av} = T_{min} \quad [\mathbf{0.05pt}] \tag{3.21}$$

Let us first obtain the minimum required thrust to fly horizontal, steady, straight flight. Multiplying and dividing by L in the second term of Equation (3.12a), and given that the aerodynamic efficiency is $E = \frac{L}{D}$, we have that:

$$T = \frac{D}{L}L = \frac{L}{E} \tag{3.22}$$

Since L is constant at those conditions of flight, the minimum required thrust occurs when the efficiency is maximum: $T_{min} \Leftrightarrow E_{max}$. **[0.05 pt]**

[3]Notice that given an altitude and a thrust setting, the aircraft can theoretically fly at two different velocities meanwhile those velocities lay between the minimum velocity (stall) and a maximum velocity (typically near the divergence velocity).

Let us now proceed deducing the maximum aerodynamic efficiency:

The aerodynamic efficiency is defined as:

$$E = \frac{L}{D} = \frac{C_L}{C_D} \tag{3.23}$$

Substituting the parabolic polar curve given in Equation (3.15) in Equation (3.23), we obtain:

$$E = \frac{C_L}{C_{D_0} + C_{D_i} C_L^2} \tag{3.24}$$

In order to seek the values corresponding to the maximum aerodynamic efficiency, one must derivate and make it equal to zero, that is:

$$\frac{dE}{dC_L} = 0 = \frac{C_{D_0} - C_{D_i} C_L^2}{(C_{D_0} + C_{D_i} C_L^2)^2} \rightarrow (C_L)_{E_{max}} = C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{C_{D_i}}}. \tag{3.25}$$

Substituting the value of $C_{L_{opt}}$ into Equation (3.24) and simplifying we obtain that:

$$E_{max} = \frac{1}{2\sqrt{C_{D_0} C_{D_i}}}. \quad [\mathbf{0.05pt}]$$

E_{max} yields 20.86, and $T_{min} = 32870$ N. **[0.05 pt]**.

According to Equation (3.17) and based on Equation (3.21) with $T_{min} = 32870$ N, we have that:

$$32870 = T_{max,av}(h = 0)\frac{\rho}{\rho_0}.$$

Given that $T_{max,av}(h = 0)$ was given in the statement and ρ_0 is known according to ISA, we have that $\rho = 0.335$ kg/m^3. **[0.05 pt]**.

Since $\rho_{h_{max}} < \rho_{11000}$ we can easily deduce that the ceiling belongs to the stratosphere. Using the ISA equation corresponding to the stratosphere we have that:

$$\rho_{h_{max}} = \rho_{11} \exp^{-\frac{g}{RT_{11}}(h_{max} - h_{11})}, \quad [\mathbf{0.05pt}]. \tag{3.26}$$

where the subindex 11 corresponds to the values at the tropopause (h=11000 m). Operating in Equation (3.26), the ceiling yields $h_{max} = 11526$ m. **[0.2 pt]**.

We want to perform a horizontal turn at an altitude of $h = 11000$ with a load factor $n = 2$, and with the velocity corresponding to the maximum aerodynamic efficiency in horizontal, steady, straight flight.

5. The required bank angle:

The equations governing the dynamics of the airplane in an horizontal turn are:

$$T = D, \quad (3.27a)$$
$$mV\dot{\chi} = L\sin\mu, \quad (3.27b)$$
$$L\cos\mu = mg, \quad [\mathbf{0.05pt}]. \quad (3.27c)$$

In a uniform (stationary) circular movement, it is well known that the tangential velocity is equal to the angular velocity ($\dot{\chi}$) multiplied by the radius of turn (R):

$$V = \dot{\chi}R. \quad [\mathbf{0.05pt}]. \quad (3.28)$$

Therefore, System (3.27) can be rewritten as:

$$T = D, \quad (3.29a)$$
$$n\sin\mu = \frac{V^2}{gR}, \quad (3.29b)$$
$$n = \frac{1}{\cos\mu}, \quad [\mathbf{0.1pt}]. \quad (3.29c)$$

where $n = \frac{L}{mg}$ is the load factor.

Therefore, looking at Equation (3.29c), it is straightforward to determine that the bank angle of turn is $\mu = 60$ [deg] **[0.1 pt]**.

6. The radius of turn.

First, we need to calculate the velocity corresponding to the maximum efficiency. As we have calculated before in Equation 3.25, the coefficient of lift that generates maximum efficiency is the so-called optimal coefficient of lift, that is, $C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{C_{D_i}}} = 0.834..$ The velocity yields then:

$$V = \sqrt{\frac{L}{\frac{1}{2}\rho S_w C_{L_{opt}}}} = 186.4 \ m/s \quad [\mathbf{0.05pt}].$$

Entering in Equation (3.29b) with $\mu = 60$ [deg] and $V = 186.4$ m/s: $R = 4093.8$ m **[0.05 pt]**.

7. The required thrust.

As exposed in Question 3., Equation (3.29a) can be expressed as:

$$T = \frac{1}{2}\rho S_w V^2 C_{D_0} + \frac{L^2}{\frac{1}{2}\rho V^2 S_w} C_{D_i}$$

Since all values are known: $T = 32451$ N. **[0.05 pt]**.

Since $T \leq T_{max,av}(h = 110000)$, the aircraft can perform the turn. **[0.05 pt]**.

8. We want to perform a horizontal turn with the same load factor and the same radius as in the previous case, but at an altitude corresponding to the theoretical ceiling of the aircraft. Can the aircraft perform the turn?

If the load factor is the same, $n = 2$, necessarily (according to Equation (3.29c)) the bank angle is the same, $\mu = 60$ [deg]. Also, if the radius of turn is the same, $R = 4093.8$ m, necessarily (according to Equation (3.29b)), the velocity of the turn must be the same as in the previous case, $V = 186.4$ m/s. Obviously, since the density will change according to the new altitude ($\rho = 0.335$ Kg/m^3), the turn will not be performed under maximum efficiency conditions. **[0.05 pt]**.

In order to know wether the turn can be performed or not, we must compare the required thrust with the maximum available thrust at the ceiling altitude:

$$T = \frac{1}{2}\rho S_w V^2 C_{D_0} + \frac{L^2}{\frac{1}{2}\rho V^2 S_w} C_{D_i} = 32983\ N\textbf{[0.05pt]}.$$

$$T_{max,av}(h = 11526) = T_{max,av}(h = 0)\frac{\rho}{\rho_0} = 32816\ N$$

Since $T > T_{max,av}$ at the ceiling, the turn can not be performed. **[0.05 pt]**.